

153. Find two integers whose sum is  $-10$  and whose product is  $-24$ .

154. Find two integers whose sum is  $-18$  and whose product is  $45$ .

### 155. The Fibonacci Sequence

The numbers in the Fibonacci sequence are  $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots$ , where each term after the second term is the sum of the two preceding terms. This famous sequence of numbers can be used to model many phenomena in nature.



(a) Form fractions of consecutive terms in the sequence. Find the decimal approximations to  $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}$ , and so on.

(b) What number do the ratios get close to? This number is called the **golden ratio** and has application in many different areas.

(c) Research Fibonacci numbers and cite three different applications.

### Explaining the Concepts

156. Write a sentence or two that justifies the fact that the product of a positive number and a negative number is a negative number. You may use an example.

157. Explain how  $42 \div 4$  may be written as a multiplication problem.

## 1.5 Adding, Subtracting, Multiplying, and Dividing Rational Numbers

### Objectives

- 1 Multiply Rational Numbers in Fractional Form
- 2 Divide Rational Numbers in Fractional Form
- 3 Add or Subtract Rational Numbers in Fractional Form
- 4 Add, Subtract, Multiply, or Divide Rational Numbers in Decimal Form

### In Words

To find the product of two or more fractions, multiply the numerators together. Then multiply the denominators together. Write the fraction in lowest terms, if necessary.

### Are You Ready for This Section?

Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

R1. Find the least common denominator of  $\frac{5}{12}$  and  $\frac{3}{16}$ . [Section 1.2, pp. 11–12]

R2. Rewrite  $\frac{4}{5}$  as an equivalent fraction with a denominator of  $30$ . [Section 1.2, pp. 10–11]

R3. Write each rational number in lowest terms. [Section 1.4, pp. 34–35]

(a)  $-\frac{18}{30}$  (b)  $-\frac{24}{4}$

Now that we are comfortable with operations on integers, we can perform operations on rational numbers. We begin with operations on rational numbers expressed as fractions and end the section with the operations on rational numbers in decimal form.

All of the properties included for integers in Section 1.4 apply to rational numbers as well. In fact, these properties apply to all real numbers.

### 1 Multiply Rational Numbers in Fractional Form

We use the following property to multiply two rational numbers in fractional form:

#### Multiplying Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad \text{where } b \text{ and } d \neq 0$$

The rules of signs that apply to integers also apply to rational numbers: The product of two positive rational numbers is positive; the product of a positive rational number and a negative rational number is negative; and the product of two negative rational numbers is positive.

Ready?...Answers R1. LCD = 48

R2.  $\frac{4}{5} = \frac{24}{30}$  R3. a.  $-\frac{3}{5}$  b.  $-6$

**EXAMPLE 1** Multiplying Rational Numbers (Fractions)Find the product:  $\frac{2}{9} \cdot \left(-\frac{15}{19}\right)$ **Solution**

We begin by rewriting the rational number  $-\frac{15}{19}$  as  $\frac{-15}{19}$ . Then we multiply the numerators and multiply the denominators.

$$\frac{2}{9} \cdot \left(\frac{-15}{19}\right) = \frac{2 \cdot (-15)}{9 \cdot 19}$$

$$\begin{array}{l} \text{Write the numerator and the denominator} \\ \text{as products of prime factors:} \end{array} = \frac{2 \cdot 3 \cdot (-5)}{3 \cdot 3 \cdot 19}$$

$$\begin{array}{l} \text{Divide out common factors:} \\ \end{array} = \frac{2 \cdot 3 \cdot (-5)}{3 \cdot 3 \cdot 19}$$

$$= \frac{2 \cdot (-5)}{3 \cdot 19}$$

$$\begin{array}{l} \text{Multiply:} \\ \end{array} = -\frac{10}{57}$$

**Work Smart**

Notice that we do not multiply in the numerator or denominator until we divide out common factors.

**Quick ✓**

In Problems 1–5, find each product, and write in lowest terms.

1.  $\frac{3}{4} \cdot \frac{9}{8}$

2.  $\frac{-5}{7} \cdot \frac{56}{15}$

3.  $\frac{12}{45} \cdot \left(-\frac{18}{20}\right)$

4.  $-\frac{25}{75} \cdot \left(-\frac{9}{4}\right)$

5.  $\frac{7}{3} \cdot \frac{1}{14} \cdot \left(-\frac{9}{11}\right)$

**2 Divide Rational Numbers in Fractional Form**

To divide rational numbers, we must know how to find the reciprocal of a rational number. In Section 1.4, we saw that two numbers are *reciprocals*, or multiplicative inverses, if their product is 1. This definition applies to any nonzero real number. Thus,  $\frac{3}{2}$  and  $\frac{2}{3}$  are reciprocals because  $\frac{3}{2} \cdot \frac{2}{3} = 1$ ; 9 and  $\frac{1}{9}$  are reciprocals because  $9 \cdot \frac{1}{9} = 1$ ;  $-\frac{4}{7}$  and  $-\frac{7}{4}$  are reciprocals because  $-\frac{4}{7} \cdot \left(-\frac{7}{4}\right) = 1$ .

**Quick ✓**

6. Two numbers are called multiplicative inverses, or reciprocals, if their product is equal to  $\underline{\hspace{1cm}}$ .

In Problems 7–10, find the reciprocal of each number.

7. 12

8.  $\frac{7}{5}$

9.  $-\frac{1}{4}$

10.  $-\frac{31}{20}$

We divide rational numbers by rewriting the division as an equivalent multiplication problem.

**Dividing Rational Numbers Expressed as Fractions**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} \quad \text{where } b, c, d \neq 0$$

**EXAMPLE 2** How to Divide Rational Numbers (Fractions)Find the quotient:  $\frac{3}{10} \div \frac{12}{25}$ **Step-by-Step Solution****Step 1:** Write the equivalent multiplication problem.

$$\frac{3}{10} \div \frac{12}{25} = \frac{3}{10} \cdot \frac{25}{12}$$

**Step 2:** Write the product in factored form and divide out common factors.

$$\begin{aligned} &= \frac{3 \cdot 25}{10 \cdot 12} \\ &= \frac{3 \cdot 5 \cdot 5}{5 \cdot 2 \cdot 4 \cdot 3} \\ &= \frac{3 \cdot 5 \cdot 5}{5 \cdot 2 \cdot 4 \cdot 3} \\ &= \frac{5}{2 \cdot 4} \end{aligned}$$

**Step 3:** Multiply the remaining factors.

$$= \frac{5}{8}$$

**Quick ✓**

In Problems 11–14, find the quotient.

11.  $\frac{5}{7} \div \frac{7}{10}$

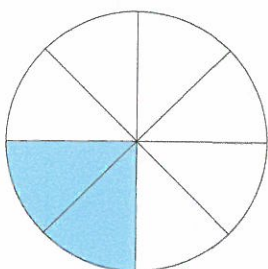
12.  $-\frac{9}{12} \div \frac{14}{7}$

13.  $\frac{8}{35} \div \left(-\frac{1}{10}\right)$

14.  $-\frac{18}{63} \div \left(-\frac{54}{35}\right)$

**3** Add or Subtract Rational Numbers in Fractional Form**Add or Subtract Rational Numbers (Fractions) with Like Denominators**

Figure 17



In Figure 17, two of the eight equal regions are shaded. Each of the regions represents the fraction  $\frac{1}{8}$ . Together, the two shaded regions make up  $\frac{1}{4}$  of the circle. This implies that

$$\begin{aligned} \frac{1}{8} + \frac{1}{8} &= \frac{1+1}{8} \\ &= \frac{2}{8} \\ &= \frac{1}{4} \end{aligned}$$

Or, suppose Bobby has \$0.25 and his grandma gives him \$0.50. He now has  $\$0.25 + \$0.50 = \$0.75$ , or  $\frac{3}{4}$  of a dollar. Because  $0.25 = \frac{1}{4}$  and  $0.50 = \frac{1}{2}$ , we can determine Bobby's good fortune using fractions:

$$\begin{aligned} \frac{1}{4} + \frac{1}{2} &= \frac{1}{4} + \frac{2}{4} \\ &= \frac{3}{4} \end{aligned}$$

Based on these results, we might conclude that to add fractions with the same denominators, we add the numerators and write the result over the common denominator.

This conclusion is correct. Also, because we can write any subtraction problem as an equivalent addition problem, we have the following methods for adding or subtracting rational numbers.

**In Words**

To add or subtract fractions with a common denominator, add or subtract the numerators and retain the denominator.

**Adding or Subtracting Rational Numbers (Fractions) with the Same Denominator**

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \text{where } c \neq 0 \quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c} = \frac{a+(-b)}{c} \quad \text{where } c \neq 0$$

**EXAMPLE 3****Adding Rational Numbers (Fractions) with the Same Denominator**

Find the sum and write in lowest terms:  $-\frac{1}{8} + \frac{3}{8}$

**Solution**

$$-\frac{1}{8} + \frac{3}{8} = \frac{-1}{8} + \frac{3}{8}$$

Write the numerators as a sum over the common denominator:  $= \frac{-1+3}{8}$

Add the numerators:  $= \frac{2}{8}$

Factor 8 and divide out the 2s:  $= \frac{1 \cdot 2}{4 \cdot 2}$   
 $= \frac{1}{4}$

**EXAMPLE 4****Subtracting Rational Numbers (Fractions) with the Same Denominator**

Find the difference and write in lowest terms:  $\frac{9}{16} - \frac{3}{16}$

**Solution**

$$\frac{9}{16} - \frac{3}{16} = \frac{9-3}{16}$$

$$= \frac{6}{16}$$

Factor 6, factor 16, and divide out the 2s:  $= \frac{3 \cdot 2}{8 \cdot 2}$

$$= \frac{3}{8}$$

**Quick ✓**

15.  $-\frac{5}{7} + \frac{3}{7} = \frac{\quad}{7} + \frac{3}{7}$

In Problems 16–19, find the sum or difference, and write in lowest terms.

16.  $\frac{8}{11} + \frac{2}{11}$

17.  $-\frac{18}{35} + \frac{3}{35}$

18.  $\frac{19}{63} - \frac{10}{63}$

19.  $-\frac{9}{10} - \frac{3}{10}$

**Work Smart: Study Skills**

Note the title of Example 5: “How to Add Rational Numbers with Unlike Denominators.” This three-column example provides a guided, step-by-step approach to solving a problem so you can see each of the steps. Cover the third column and try to work the example yourself. Then look at the entries in the third column and check your solution. Was it correct?

### ▶ Adding or Subtracting Rational Numbers (Fractions) with Unlike Denominators

How do we add rational numbers with different denominators? We must find the least common denominator of the two rational numbers. Recall from Section 1.2 that the **least common denominator (LCD)** is the smallest number that each denominator has as a common multiple.

#### EXAMPLE 5 How to Add Rational Numbers (Fractions) with Unlike Denominators

Find the sum:  $\frac{5}{6} + \frac{3}{8}$

##### Step-by-Step Solution

**Step 1:** Find the least common denominator of the fractions.

Write each denominator as the product of prime factors, arranging like factors vertically:

$$\begin{array}{r} 6 = 2 \quad \cdot 3 \\ 8 = 2 \cdot 2 \cdot 2 \end{array}$$

Find the product of each of the prime factors the greatest number of times they appear in any factorization:

$$\begin{array}{r} \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \\ = 24 \end{array}$$

**Step 2:** Write each rational number with the denominator found in Step 1.

Use  $1 = \frac{4}{4}$  to change the denominator 6 to 24, and use  $1 = \frac{3}{3}$  to change the denominator 8 to 24:

$$\begin{aligned} \frac{5}{6} + \frac{3}{8} &= \frac{5}{6} \cdot \frac{4}{4} + \frac{3}{8} \cdot \frac{3}{3} \\ &= \frac{20}{24} + \frac{9}{24} \end{aligned}$$

**Step 3:** Add the numerators and write the result over the common denominator.

$$\begin{aligned} &= \frac{20 + 9}{24} \\ &= \frac{29}{24} \end{aligned}$$

**Step 4:** Write in lowest terms.

The rational number is in lowest terms, so  $\frac{5}{6} + \frac{3}{8} = \frac{29}{24}$ .

#### EXAMPLE 6 How to Subtract Rational Numbers (Fractions) with Unlike Denominators

Find the difference:  $-\frac{9}{14} - \frac{1}{6}$

##### Step-by-Step Solution

**Step 1:** Find the least common denominator of the fractions.

Write each denominator as the product of prime factors, aligning like factors vertically:

$$\begin{array}{r} 14 = 2 \cdot 7 \\ 6 = 2 \quad \cdot 3 \end{array}$$

Find the product of each of the prime factors the greatest number of times it appears in any factorization:

$$\begin{array}{r} \text{LCD} = 2 \cdot 7 \cdot 3 \\ = 42 \end{array}$$

**Step 2:** Write each rational number with the denominator found in Step 1.

Use  $1 = \frac{3}{3}$  to change the denominator 14 to 42, and use  $1 = \frac{7}{7}$  to change the denominator 6 to 42:

$$\begin{aligned} -\frac{9}{14} - \frac{1}{6} &= -\frac{9}{14} \cdot \frac{3}{3} - \frac{1}{6} \cdot \frac{7}{7} \\ &= -\frac{27}{42} - \frac{7}{42} \end{aligned}$$

(continued)

**Step 3:** Subtract the numerators and write the result over the common denominator.

$$\begin{aligned} &= \frac{-27 - 7}{42} \\ &= \frac{-34}{42} \end{aligned}$$

**Step 4:** Write in lowest terms.

Factor  $-34$  and  $42$  and divide out like factors:

$$\begin{aligned} &= \frac{2 \cdot (-17)}{2 \cdot 21} \\ &= \frac{-17}{21} \end{aligned}$$

### Adding or Subtracting Rational Numbers (Fractions) with Unlike Denominators

**Step 1:** Find the LCD of the rational numbers.

**Step 2:** Write each rational number with the LCD.

**Step 3:** Add or subtract the numerators and write the result over the common denominator.

**Step 4:** Write the result in lowest terms.

#### Quick ✓

In Problems 20–23, find each sum or difference, and write in lowest terms.

20.  $\frac{3}{14} + \frac{10}{21}$

21.  $\frac{5}{12} - \frac{5}{18}$

22.  $-\frac{23}{6} + \frac{7}{12}$

23.  $-\frac{1}{15} - \frac{1}{12}$

Remember, the direction “evaluate” means to find the value of the expression.

### EXAMPLE 7

### Evaluating an Expression Containing Rational Numbers

Evaluate and write in lowest terms:  $4 - \frac{2}{3}$

#### Solution

The key is to remember that  $4 = \frac{4}{1}$ .

$$4 - \frac{2}{3} = \frac{4}{1} - \frac{2}{3}$$

Rewrite each fraction with LCD = 3:  $= \frac{4 \cdot 3}{1 \cdot 3} - \frac{2}{3}$

$$= \frac{12}{3} - \frac{2}{3}$$

$$= \frac{10}{3}$$

#### Quick ✓

In Problem 24 and 25, evaluate and write in lowest terms.

24.  $-2 + \frac{7}{16}$

25.  $6 - \frac{9}{4}$

#### 4 Add, Subtract, Multiply, or Divide Rational Numbers in Decimal Form

##### ▶ Adding or Subtracting Decimals

To add or subtract decimals, arrange the numbers in a column with the decimal points aligned. Then add or subtract the digits in the like place values. Put the decimal point in the answer directly below the decimal points in the problem.

#### EXAMPLE 8 Adding or Subtracting Decimals That Have the Same Sign

Evaluate each expression:

(a)  $2.93 + 7.2 + 3.026$       (b)  $76.4 - 4.95$

##### Solution

(a) Use zeros as placeholders:

$$\begin{array}{r} 2.930 \\ 7.200 \\ +3.026 \\ \hline 13.156 \end{array}$$

(b) Use zero as a placeholder:

$$\begin{array}{r} 76.40 \\ -4.95 \\ \hline 71.45 \end{array}$$

##### Work Smart

A whole number has an implied decimal point. For example,  $74 = 74.000$

#### EXAMPLE 9 Adding or Subtracting Decimals That Have Different Signs

Evaluate each expression:

(a)  $100.32 - (-32.015)$       (b)  $-23.03 + 18.49$

##### Solution

(a) Remember,  $a - (-b) = a + b$ , so  $100.32 - (-32.015) = 100.32 + 32.015$ .

$$\begin{array}{r} 100.320 \\ +32.015 \\ \hline 132.335 \end{array}$$

(b) Recall that to add real numbers with different signs, subtract the smaller absolute value from the larger absolute value and attach the sign of the larger absolute value. Because  $|-23.03| = 23.03$  and  $|18.49| = 18.49$ , we compute  $23.03 - 18.49$  and attach a negative sign to the difference.

$$\begin{array}{r} 23.03 \\ -18.49 \\ \hline 4.54 \end{array}$$

So,  $-23.03 + 18.49 = -4.54$ .

##### Quick ✓

In Problems 26–29, find the sum or difference.

26.  $9.67 - (-11.344)$

27.  $-17.39 + 81.96$

28.  $-74.28 + 14.832$

29.  $-180.782 + 100.3 + 9.07$

### ▶ Multiplying Decimals

The rules for multiplying decimals come from the rules for multiplying rational numbers in fractional form. For example,

$$\underbrace{-0.7}_{\substack{1 \text{ decimal} \\ \text{place}}} \cdot \underbrace{0.03}_{\substack{2 \text{ decimal} \\ \text{places}}} = \frac{-7}{10} \cdot \frac{3}{100} = \frac{-21}{1000} = \underbrace{-0.021}_{\substack{3 \text{ decimal} \\ \text{places}}}$$

Notice the number of digits to the right of the decimal point in the answer is equal to the sum of the numbers of digits to the right of each decimal point in the factors.

The rules of signs that we learned in Section 1.4 apply to decimals as well.

#### EXAMPLE 10 Multiplying Decimals

Find the product:

(a)  $3.43 \cdot 2.6$       (b)  $-3.17 \cdot 0.02$

**Solution**

(a)

$$\begin{array}{r} 3.43 \\ \times 2.6 \\ \hline 2058 \\ 686 \\ \hline 8.918 \end{array}$$

Annotations for (a):  
 - "two digits to the right of the decimal point" points to 3.43.  
 - "one digit to the right of the decimal point" points to 2.6.  
 - "three digits to the right of the decimal point" points to 8.918.

(b)

$$\begin{array}{r} -3.17 \\ \times 0.02 \\ \hline -0.0634 \end{array}$$

Annotations for (b):  
 - "two digits to the right of the decimal point" points to 0.02.  
 - "four digits to the right of the decimal point" points to -0.0634.  
 - "two digits to the right of the decimal point" points to -3.17.

#### Work Smart

The number of digits to the right of the decimal point in the product is the *sum* of the numbers of digits to the right of each decimal point in the factors.

#### Multiplying Decimals

**Step 1:** Multiply the factors as if they were whole numbers.

**Step 2:** Place the decimal point so the number of digits to the right of the decimal point in the product equals the *sum* of the numbers of digits to the right of each decimal point in the factors.

#### Quick ✓

In Problems 30–33, find the product.

30.  $23.9 \cdot 0.2$

31.  $257 \cdot (-3.5)$

32.  $-3.45 \cdot 0.03$

33.  $-0.03 \cdot (-0.45)$

### ▶ Dividing Decimals

In the division problem  $2 \overline{)7.94}$ , the number 2 is the *divisor*, 7.94 is the *dividend*, and 3.97 is the *quotient*. Notice that when we divide by a whole number, we line up the decimal



points in the quotient and the dividend. In algebra, we typically write this division problem as  $\frac{7.94}{2} = 3.97$ .

To divide decimals, we want the divisor to be a whole number, so we multiply the dividend and the divisor by a power of 10 that will make the divisor a whole number. Then we divide as described above.

### EXAMPLE 11 Dividing Decimals

(a) Divide:  $\frac{22.26}{15.9}$

(b) Divide:  $\frac{0.03724}{-0.38}$

#### Solution

(a) Since the divisor 15.9 is fifteen and nine-tenths, we multiply  $\frac{22.26}{15.9}$  by  $\frac{10}{10}$  to make the divisor a whole number and obtain  $\frac{22.26}{15.9} \cdot \frac{10}{10} = \frac{222.6}{159}$ . Now we divide.

$$\begin{array}{r} 1.4 \\ 159 \overline{)222.6} \\ \underline{159} \phantom{.6} \\ 636 \\ \underline{636} \\ 0 \end{array}$$

So  $\frac{22.26}{15.9} = 1.4$ .

(b) Because we have a positive number divided by a negative number, the quotient will be negative. The divisor 0.38 is thirty-eight hundredths, so we multiply  $\frac{0.03724}{0.38}$  by  $\frac{100}{100}$  to make the divisor a whole number and obtain  $\frac{0.03724}{0.38} \cdot \frac{100}{100} = \frac{3.724}{38}$ . Now we divide.

$$\begin{array}{r} 0.098 \\ 38 \overline{)3.724} \\ \underline{342} \phantom{.4} \\ 304 \\ \underline{304} \\ 0 \end{array}$$

Therefore,  $\frac{0.03724}{-0.38} = -0.098$ .

#### In Words

To divide decimals, change the divisor to a whole number and divide.

#### Dividing Decimals

**Step 1:** Multiply the dividend and divisor by a power of 10 that will make the divisor a whole number.

**Step 2:** Divide as though working with whole numbers.

#### Work Smart: Study Skills

Selected problems in the exercise sets are identified by green color. For extra help, worked solutions to these problems are in MyMathLab.

#### Quick ✓

In Problems 34–36, find the quotient.

34.  $\frac{18.25}{73}$

35.  $\frac{1.0032}{0.12}$

36.  $\frac{-4.2958}{45.7}$

## 1.5 Exercises

MyMathLab<sup>®</sup>

Exercise numbers in green  
have complete video solutions  
in MyMathLab.

Problems 1–36 are the QuickChecks that follow the EXAMPLES.

## Building Skills

In Problems 37–42, write each rational number in lowest terms.

$$\begin{array}{lll} 37. \frac{14}{21} & 38. \frac{9}{15} & 39. -\frac{38}{18} \\ 40. -\frac{81}{36} & 41. -\frac{22}{44} & 42. -\frac{24}{27} \end{array}$$

In Problems 43–52, find the product and write in lowest terms. See Objective 1.

$$\begin{array}{lll} 43. \frac{6}{5} \cdot \frac{2}{5} & 44. \frac{7}{8} \cdot \frac{10}{21} & 45. -\frac{5}{2} \cdot 10 \\ 46. -\frac{3}{7} \cdot 63 & 47. -\frac{3}{2} \cdot \frac{4}{9} & 48. -\frac{5}{2} \cdot \frac{16}{25} \\ 49. -\frac{22}{3} \cdot \left(-\frac{12}{11}\right) & 50. -\frac{60}{75} \cdot \left(-\frac{25}{4}\right) & \\ 51. \frac{3}{16} \cdot \frac{8}{9} & 52. \frac{4}{27} \cdot \frac{9}{16} & \end{array}$$

In Problems 53–56, find the reciprocal of each number. See Objective 2.

$$\begin{array}{ll} 53. \frac{3}{5} & 54. \frac{9}{4} \\ 55. -5 & 56. -8 \end{array}$$

In Problems 57–66, find the quotient and write in lowest terms. See Objective 2.

$$\begin{array}{ll} 57. \frac{4}{9} \div \frac{8}{15} & 58. \frac{3}{2} \div \frac{9}{8} \\ 59. -\frac{1}{3} \div 3 & 60. -\frac{1}{4} \div 4 \\ 61. \frac{5}{6} \div \left(-\frac{5}{4}\right) & 62. \frac{4}{3} \div \left(-\frac{9}{10}\right) \\ 63. \frac{2}{5} \div \frac{22}{5} & 64. \frac{44}{63} \div \frac{88}{21} \\ 65. -8 \div \left(-\frac{1}{4}\right) & 66. -3 \div \left(-\frac{1}{6}\right) \end{array}$$

In Problems 67–84, find the sum or difference and write in lowest terms. See Objective 3.

$$\begin{array}{ll} 67. \frac{3}{4} + \frac{3}{4} & 68. \frac{6}{11} + \frac{16}{11} \\ 69. \frac{9}{8} - \frac{5}{8} & 70. \frac{12}{5} - \frac{2}{5} \\ 71. \frac{6}{7} - \left(-\frac{8}{7}\right) & 72. \frac{2}{3} - \left(-\frac{7}{3}\right) \\ 73. -\frac{5}{3} + 2 & 74. -\frac{7}{8} + 4 \\ 75. 6 - \frac{7}{2} & 76. 3 - \frac{5}{3} \end{array}$$

$$77. -\frac{4}{3} + \frac{1}{4}$$

$$79. \frac{7}{5} + \left(-\frac{23}{20}\right)$$

$$81. \frac{8}{15} - \frac{7}{10}$$

$$83. -\frac{33}{10} - \left(-\frac{33}{8}\right)$$

$$78. -\frac{2}{5} + \left(-\frac{2}{3}\right)$$

$$80. \frac{7}{15} - \left(-\frac{4}{3}\right)$$

$$82. \frac{17}{6} - \frac{13}{9}$$

$$84. -\frac{29}{6} - \left(-\frac{29}{20}\right)$$

In Problems 85–102, perform the indicated operation(s). See Objective 4.

$$85. -10.5 + 4$$

$$87. -(-3.5) + 4.9$$

$$89. 39.1 - (-16.82)$$

$$91. -5.21 - (-6.7)$$

$$93. 45 - 2.45$$

$$95. 4.3 \cdot 5.8$$

$$97. 0.075 \cdot (-120)$$

$$99. \frac{136.08}{5.6}$$

$$101. \frac{-25.48}{0.052}$$

$$86. -13.2 + 7$$

$$88. -(-32.9) + 10.3$$

$$90. 29.23 - (-12.98)$$

$$92. -4.94 - (-3.87)$$

$$94. 32 - 5.68$$

$$96. 3.1 \cdot 10.9$$

$$98. 0.065 \cdot (-340)$$

$$100. \frac{332.59}{7.9}$$

$$102. \frac{-48}{0.03}$$

## Mixed Practice

In Problems 103–134, evaluate and write in lowest terms.

$$103. -\frac{5}{6} + \frac{7}{15}$$

$$105. -\frac{10}{21} \cdot \frac{14}{5}$$

$$107. \frac{3}{64} \div \left(-\frac{9}{16}\right)$$

$$109. -\frac{5}{12} + \frac{2}{12}$$

$$111. -\frac{2}{7} - \frac{17}{5}$$

$$113. -8.7 - (-10.3)$$

$$115. \frac{1}{12} + \left(-\frac{5}{28}\right)$$

$$117. -12.03 \cdot 4.2$$

$$119. 36 \cdot \left(-\frac{4}{9}\right)$$

$$121. -27 \div \frac{9}{5}$$

$$123. 3.62 - 10.2$$

$$104. -\frac{8}{9} + \left(-\frac{16}{21}\right)$$

$$106. \frac{24}{5} \cdot \left(-\frac{35}{4}\right)$$

$$108. -\frac{12}{7} \div \left(-\frac{4}{21}\right)$$

$$110. -\frac{4}{9} + \frac{1}{9}$$

$$112. -\frac{3}{4} - \frac{1}{5}$$

$$114. -4.63 - (-12.9)$$

$$116. \frac{3}{16} + \left(-\frac{7}{40}\right)$$

$$118. 34.2 \cdot (-8.43)$$

$$120. -\frac{8}{3} \cdot 15$$

$$122. -24 \div \frac{8}{7}$$

$$124. 4.75 - 6.2$$

125.  $\frac{-145.518}{18.42}$

126.  $\frac{-297.078}{22.17}$

127.  $\frac{12}{7} - \frac{17}{14} - \frac{48}{21}$

128.  $\frac{9}{4} - \frac{21}{6} - \frac{11}{8}$

129.  $54.2 - 18.78 - (-2.5) + 20.47$

130.  $90.3 - 100.9 - (-34.26) + 32.95$

131.  $(400)(-25.8)(0.003)$

132.  $(500)(-12.4)(-0.02)$

133.  $-\frac{11}{12} - \left(-\frac{1}{6}\right) + \frac{7}{8}$

134.  $\frac{8}{15} - \left(-\frac{7}{9}\right) + \frac{2}{3}$

**Applying the Concepts**

**135. Watching TV** If Rachel spends  $\frac{1}{8}$  of her life watching TV, how many hours of TV does she watch in one week?

**136. Halloween Candy** Henry decided to make  $\frac{2}{3}$ -oz bags of candy for treats at Halloween. If he bought 16 oz of candy, how many bags will he have to give away?

**137. Biology Class** Susan's biology class begins with 36 students. If  $\frac{2}{3}$  will finish the course and  $\frac{3}{4}$  of those get a passing grade, how many students will pass Susan's biology class this term?

**138. Pizza Time** Joyce and Ramie bought a pizza. Joyce ate  $\frac{2}{5}$  of the pizza and Ramie ate  $\frac{1}{9}$  of what was left. What fraction of the pizza remains uneaten?

**139. Hourly Pay** Last week, Jonathon received a paycheck for \$442.80. The withholding for federal, state, and FICA (social security and Medicare) taxes was \$97.20. If Jonathon worked 30 hours last week, what is his hourly pay rate?

**140. Average Revenue** Aqsa runs an online store selling beauty products. Last year, the total revenue of the company was \$29,409.12. What was the average revenue per month?

**141. Stock Prices** The price per share of Intel stock has been up and down lately. On Monday it rose 2.75; on Tuesday it rose 0.87; on Wednesday it dropped 1.12; on Thursday it rose 0.52; and on Friday it fell 0.62. What was the net change in Intel's stock price per share for the week?

**142. Bank Balance** Henry started the month with \$43.68 in his checking account. During the month the following transactions occurred: He deposited his paycheck of \$929.30; and he wrote checks for rent \$650, phone \$33.49, credit card \$229.50, cable service \$75.50, and groceries \$159.30. How much does he have in his account now?

**Extending the Concepts**

Problems 143–146 use the following definition.

If  $P$  and  $Q$  are two points on a real number line with coordinates  $a$  and  $b$ , respectively, then the **distance between  $P$  and  $Q$** , denoted by  $d(P, Q)$ , is

$$d(P, Q) = |b - a|$$

**143.** Find the distance between the points  $P$  and  $Q$  on the real number line if  $P = -9.7$  and  $Q = 3.5$ .

**144.** Find the distance between the points  $P$  and  $Q$  on the real number line if  $P = -12.5$  and  $Q = 2.6$ .

**145.** Find the distance between the points  $P$  and  $Q$  on the real number line if  $P = -\frac{13}{3}$  and  $Q = \frac{7}{5}$ .

**146.** Find the distance between the points  $P$  and  $Q$  on the real number line if  $P = -\frac{5}{6}$  and  $Q = 4$ .

**Explaining the Concepts**

**147.** We know that 6 divided by 2 is 3. Explain why 6 divided by  $\frac{1}{2}$  is 12.

**148.** Use a figure like Figure 17 on page 39 to explain why  $\frac{1}{6} + \frac{2}{6} = \frac{1}{2}$ .

## Putting the Concepts Together (Sections 1.2–1.5)

We designed these problems so that you can review Sections 1.2–1.5 and show your mastery of the concepts. Take time to work these problems before proceeding with the next section. The answers to these problems are located at the back of the text on page AN-2.

- Write  $\frac{7}{8}$  and  $\frac{9}{20}$  as equivalent fractions with the least common denominator.
- Write  $\frac{21}{63}$  as a fraction in lowest terms.
- Convert  $\frac{2}{7}$  to a decimal.
- Write 0.375 as a fraction in lowest terms.
- Write 12.3% as a decimal.
- Write 0.0625 as a percent.
- Use the set  $\left\{-12, -\frac{14}{7}, -1.25, 0, \sqrt{2}, 3, 11.2\right\}$  to list all of the elements that are:
  - integers
  - rational numbers
  - irrational numbers
  - real numbers
- Replace the ? with the correct symbol  $>$ ,  $<$ ,  $=$ :  $\frac{1}{8} ? 0.5$

In Problems 9–30, perform the indicated operation and write in lowest terms.

- $17 + (-28)$
- $18 - 45$
- $-18 - (-12.5)$
- $25(-4)$
- $\frac{-35}{7}$
- $27 \div -3$
- $7 - \frac{4}{5}$
- $-\frac{5}{12} - \frac{1}{18}$
- $\frac{2}{7} \div (-8)$
- $3.56 - (-7.2)$
- $62.488 \div 42.8$
- $-23 + (-42)$
- $3 - (-24)$
- $(-5)(2)$
- $(-8)(-9)$
- $\frac{-32}{-2}$
- $-\frac{4}{5} - \frac{11}{5}$
- $\frac{7}{12} + \frac{5}{18}$
- $\frac{6}{25} \cdot 15 \cdot \frac{1}{2}$
- $\frac{0}{-8}$
- $18.946 - 11.3$
- $(7.94)(2.8)$

## 1.6 Properties of Real Numbers

### Objectives

- Use the Identity Properties of Addition and Multiplication
- Use the Commutative Properties of Addition and Multiplication
- Use the Associative Properties of Addition and Multiplication
- Understand the Multiplication and Division Properties of 0

### Are You Ready for This Section?

Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

- R1.** Find the sum:  $12 + 3 + (-12)$  [Section 1.4, pp. 28–30]
- R2.** Find the product:  $\frac{3}{4} \cdot 11 \cdot \frac{4}{3}$  [Section 1.5, pp. 37–38]

This section presents properties of real numbers. A property in mathematics is a rule that is always true. These properties will be used throughout this text and in future math courses, so it is extremely important that you understand these properties and know how to use them.

### 1 Use the Identity Properties of Addition and Multiplication

The real number 0 is the only number that when added to any real number  $a$  results in the same real number  $a$ .